

MATHEMATICAL MODELING OF CATTLE GRAZING IN MEADOWS WITH DOMINATING FESTUCA RUPICOLA IN TALISH MOUNTAINS OF AZERBAIJAN

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Abstract. Protecting of biodiversity are the main problem of our nowadays. In Talish Mountains meadows consist from different plant communities, such as, motley grass with dominating *Festuca rupicola*, plant communities from *Festuca rupicola*, *Thymus trautvetteri*, *Astracantha euoplus*. There are exist also plant communities of *Festuca alaiica* Drob., *Actracantha euoplus* Trautv., *Acantholimon hohenackeri* (Janb.et.Spach) Boiss, and plant communities of *Zerna variegata* (Bieb.) Holub., *Plantago lanceolata* L, *Thymus hohenackeri*. Meadow plants often respond differently to grazing of varying intensity. Were proposed new mathematical model for the describing cattle grazing in meadows of Talish Mountains. The analysis of model shows that the continuous grazing of six cattle per hectare where three of them are grazing on *Festuca rupicola*, will have devastating effects on grass covering of meadows territory, so that after 47 days of continuous grazing farmers should find new territories for the cattle grazing, because after 47 days continuous grazing the grasses covering of meadow territory will reduce from 70% to 29.56%. On the other hand, cattle grazing regime when one cattle are feeding on *Festuca rupicola*, but other five animals are feeding by *Thymus trautvetteri*, *Astracantha euoplus*, has less devastating effect on meadow ecosystems with dominating *Festuca rupicola*. In this case farmers after 80-90 continuous grazing will have to find new meadows for the cattle grazing.

Keywords: *mathematical modeling, cattle grazing, meadow ecosystems.*

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1. Introduction

Meadow plants often respond differently to grazing of varying intensity. For some species it acts negatively, and with increasing intensity of grazing plants began reduce their participation in the herbage until the complete disappearance. Many species respond to grazing positively up to a certain limit and then show a negative reaction. Only a few species can grow in heavy grazing conditions and trampling by livestock [1]. On the other hand, farmers need information on the potential and sustainability of pasture by the cattle grazing.

In this paper our aims is examine cattle grazing in meadow of Talish Mountains, compare it continuous grazing through mathematical models.

The questions of mathematical modeling of cattle grazing widely covered in literature. Indicates that the unlimited cattle grazing leads to the devastation of meadow areas [2, 3, 4, 5].

Protecting of biodiversity are the main problem of our nowadays. In Talish Mountains meadows consist from different plant communities, such as, motley grass with dominating *Festuca rupicola*, plant communities from *Festuca rupicola*, *Thymus*

trautvetteri, *Astracantha euoplus*. There are exist also plant communities of *Festuca alaiica* Drob., *Actracantha euoplus* Trautv., *Acantholimon hohenackeri* (Janb.et.Spach) Boiss, and plant communities of *Zerna variegata* (Bieb.) Holub., *Plantago lanceolata* L., *Thymus hohenackeri*.

Cattle grazing of grasses are the one of the main factors which makes changes in the species composition of grassland communities of Talish Mountains in Lenkoran region of Azerbaijan Republic. Now in those territories approximately 8391 hectares are used as grazing territories [7].

2. Model and results

Our models of cattle grazing are based on Verhulst exponential growth model [8]:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right) \quad (1)$$

In this model N is the number of population, r is per capita birth rate, K is the carrying capacity of the environment, which is determined by the amount of available resources. According this model without cattle grazing plant population in meadow will grow exponentially until reached the volume of the carrying capacity.

If $N(0) = N_0$ the solution of (1) is

$$\frac{N_0 K e^{rt}}{[K + N_0(e^{rt} - 1)]} \rightarrow K \quad \text{as } t \rightarrow \infty \quad [9] \quad (2)$$

Eaten part of the plant was simulated in the form of the expression:

$$G(t) = h - m(n - x)^2, \quad (3)$$

where h is a total number cows grazing in the 1 hectare studied territory, (n) covering percentage of *Festuca rupicola*, m is a number of cows grazing by *Festuca rupicola*.

Therefore, the proposed model of cattle grazing can be expressed as a equation:

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) - [h - m(n - x)^2] \quad (4)$$

In the proposed model as an independent variable we have used the degree of coverage the territory by meadow grasses, the value which measured by number plants per hectare, or by percentage, the time (t) is measured by days.

In Talish mountain grasses community from *Festuca rupicola*, *Thymus trautvetteri* and *Astracantha euoplus* are covering about 70 % of the meadows territory. The covering percentage of *Festuca rupicola* approximately equals to 30%. The tiller increment of *Festuca rupicola* is approximately equal to 1 sm per day. In the 8391 hectares of meadows of Talish Mountains are grazing about 53518 cows, therefore as a (h) we can take the approximately 6 cows per hectare. From this number about 3 cows are grazing by eating *Festuca rupicola*. By using this dates the equation (4) can be rewritten as a expression:

$$\frac{dx}{dt} = x \left(1 - \frac{x}{70}\right) - [6 - 3(30 - x)^2] \quad (5)$$

with initial condition

$$x(0)=70 \tag{6}$$

Slope field of differential equation (5) are shown in Figure 1.

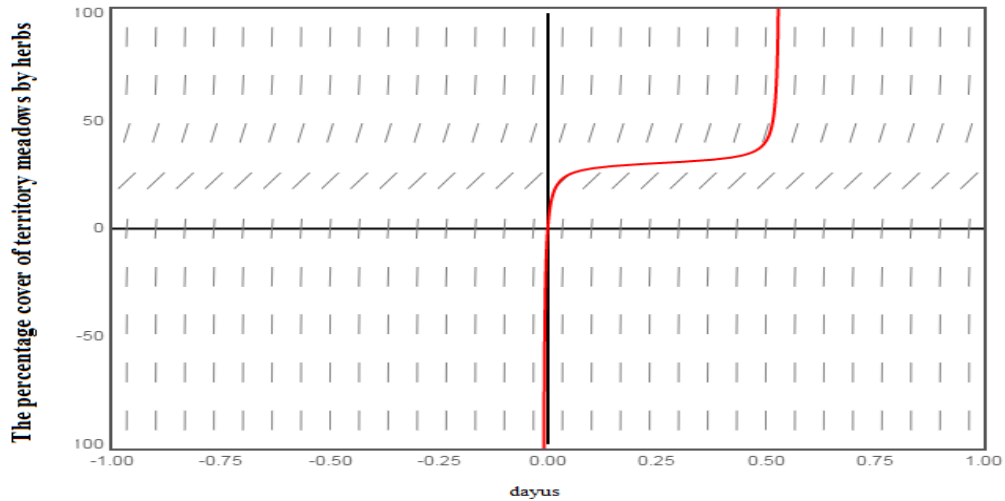


Fig. 1. The slope field of differential equation $\frac{dx}{dt} = x\left(1 - \frac{x}{70}\right) - [6 - 3(30 - x)^2]$

According to Fig.1 there is exist a weak positive slope of grasses covering meadows territory during first 50 days cattle grazing , which is meant that six cow per hectare has great affect on the tiller growth of grasses.

The solution of equation (5) can be expressed as follows:

$$x(t) = \frac{1}{209} \left\{ \sqrt{162995} \tan \left[\frac{1}{70} \left(\sqrt{162995} x_1 + \sqrt{162995} t \right) \right] + 6265 \right\} \tag{7}$$

We find (c_1) by using initial condition (6):

$$c_1 = -37.06$$

Then the equation 7 can be rewritten in the following way

$$x(t) = \frac{1}{209} [403,73 \tan(-213.84 + 5.77t) + 6265] \tag{8}$$

By using equation (8) we first want to get answer for the question, 6 cattle per hectare grazing how long in days farmers can use the meadow territory for the grazing? To the answer for this question it is necessary find the coordinates of intersection equation (8) with horizontal line $y=0$.

$$\frac{1}{209} [403,73 \tan(-213.84 + 5.77t) + 62.65] = 0 \Rightarrow t = 47,15$$

days.

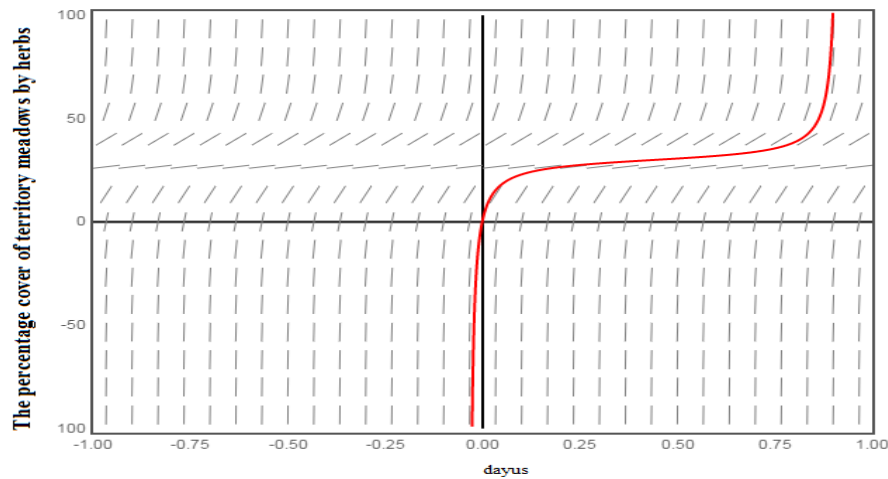
Now we want to know, after grazing how meadow territory will change. To get answer for this question we should to solve equations.

$$x(0) = -\frac{40373}{20900} \tan\left(\frac{5346}{25}\right) + \frac{6265}{209} = 29.56$$

This means that after continuous cattle grazing the grasses covering meadow territory will reduce from 70% to 29.56%.

Arise the question, how the changes of cattle grazing regime on *Festuca rupicola* will affect on grasses covering on meadow territory? It is clear that the number of cattle grazing on *Festuca rupicola* can varied from 1 to 6 animals per hectare (Fig 3).

a)



b)

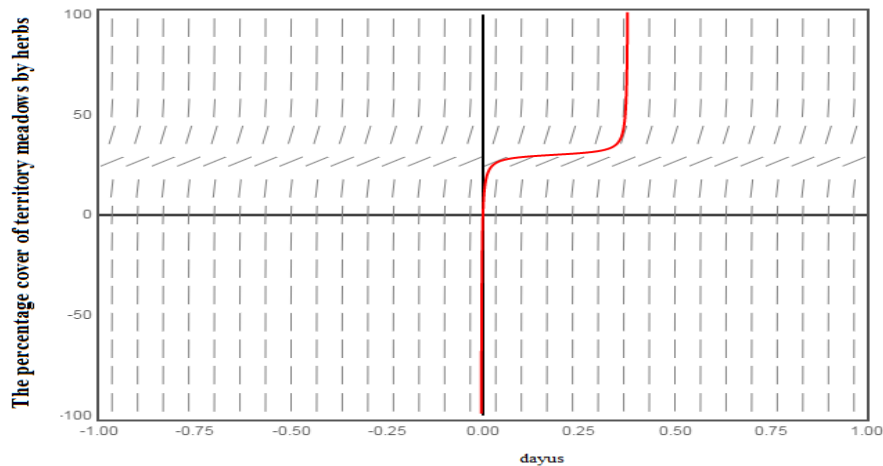


Fig 2. Impact of grazing regime on covering degree of *Festuca rupicola* of meadow territory: a) one cattle grazing; b) six cattle grazing

As is shown in Figure 2a, in the case of 1 cattle grazing on *Festuca rupicola* there exist a weak positive slope, but in the case 6 cattle per hectare there is almost no any positive dynamics of grasses covering of meadow territory (Fig 2b).

3. Conclusion

The model shows that the under mentioned above conditions the meadow grasses with dominating *Festuca rupicola* has no ability to feed the 6 cattle per hectare. If 3 cattle will fed on *Festuca rupicola*, but other 3 animals feeding on *Thymus trautvetteri*

and *Astracantha euoplus*, then farmers after approximately 50 days should find the new meadow territories for the cattle grazing, because after 47 days continuous grazing the grasses covering of meadow territory will reduce from 70% to 29,56%.

If one cattle fed on *Festuca rupicola*, but other 5 animals by *Thymus trautvetteri* and *Astracantha euoplus*, then after approximately 80-90 days farmers should find new meadows for cattle grazing. And if all of 6 animals fed on *Festuca rupicola*, then after 30 days grazing the meadow grass will be devastated.

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